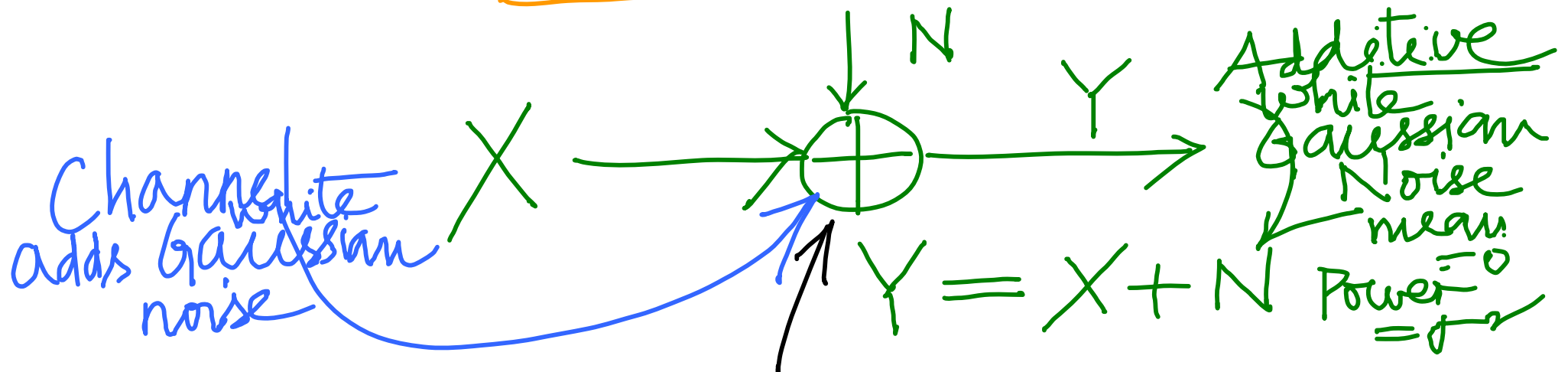




CAPACITY OF GAUSSIAN CHANNEL



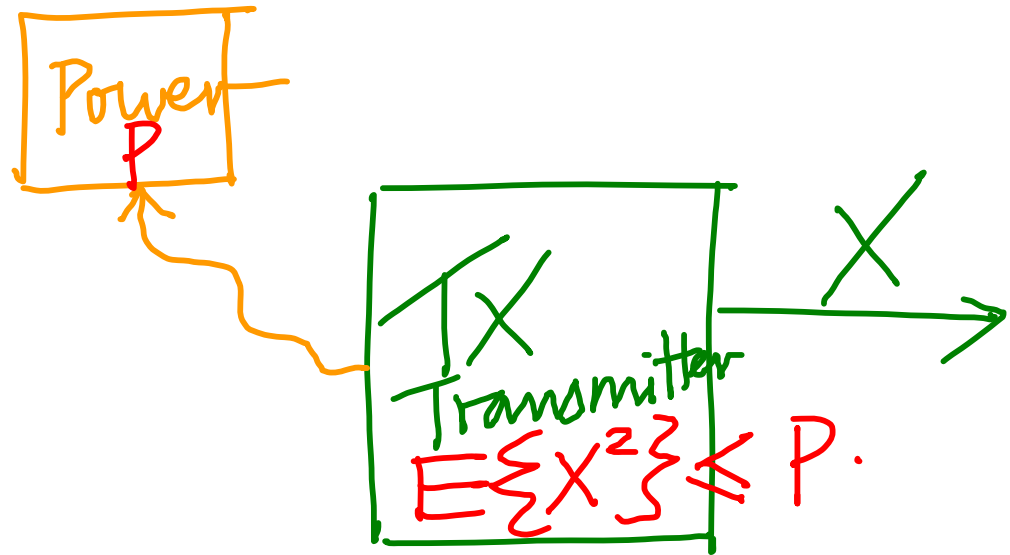
↑
one of the most
Frequently encountered
& Practically relevant
comm. channel models.

Capacity: Maximum rate of
information Transfer

At arbitrarily low
probability of Error

$$C = \max_{f_X(x): \underline{E\{X^2\}} \leq P} I(X; Y)$$

over input Probability Density Functions



Maximize Channel Capacity: Subject to Power constraint

$$\begin{aligned}
 I(X; Y) &= h(X) - h(X|Y) \\
 &= \underbrace{h(Y) - h(Y|X)}_{\text{convenient}}
 \end{aligned}$$

$$Y = X + N$$

Gaussian
mean = 0
var = σ^2

Given X

$Y = \text{Gaussian}$
mean = X
var = σ^2

$X + N$ simply shifts mean
variance remains
unchanged.

$$Y|X \sim \mathcal{N}(X, \sigma^2)$$

Gaussian

$$h(Y|X) = h(\mathcal{N}(X, \sigma^2)) \\ = \frac{1}{2} \log_2(2\pi\sigma^2 e)$$

Does NOT depend
on mean X

$$I(X; Y) = h(Y) - h(Y|X)$$

$$= h(Y) - \frac{1}{2} \log_2 (2\pi\sigma^2 e)$$

mean = 0

$$Y = X + N$$

Signal X , Noise N
are independent

$$E\{XN\} = \underbrace{E\{X\}}_0 \cdot \underbrace{E\{N\}}_0$$

$$= 0$$

✓ $\Rightarrow$ Signal & Noise
are uncorrelated.

$$\begin{aligned} E\{Y\} &= E\{X\} + E\{N\} \\ &= 0 + 0 = 0. \end{aligned}$$

$$\begin{aligned} E\{Y^2\} &= E\{(X+N)^2\} \\ &= E\{X^2 + N^2 + 2XN\} \\ &= \underbrace{E\{X^2\}}_{\text{K.P.}} + E\{N^2\} + \underbrace{2E\{XN\}}_{0}. \end{aligned}$$

$$E\{Y^2\} \leq \frac{P + \sigma^2}{\tilde{\sigma}^2}$$

$$E\{Y^2\} \leq \tilde{\sigma}^2$$

$$C = \max_{f_Y(y): E\{Y^2\} \leq \tilde{\sigma}^2} I(X; Y)$$

constant does NOT depend on $f_Y(y)$

$$= \max_{f_Y(y): E\{Y^2\} \leq \tilde{\sigma}^2} h(Y) - \frac{1}{2} \log_2(2\pi\tilde{\sigma}^2 e)$$

$$= \left(\max_{f_Y(y): E\{Y^2\} \leq \sigma^2} h(Y) \right)$$

$$= \underbrace{-\frac{1}{2} \log(2\pi\sigma^2 e)}_{\text{constant}}$$

only $h(Y)$ needs to be maximized wrt to PDF of Y .